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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

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In 1903[1] I published an account of experiments which I had made with falling bronze spheres, one inch in diameter, in the tower of the Jefferson Physical Laboratory. The especial object of these experiments was to look for a southerly deviation, from the plumb line vertical, of the course of the falling balls, several observers, from the time of Hooke, 1680, to Rundell, 1848, having reported finding such a deviation, though Gauss and Laplace, both of whom discussed the matter theoretically about 1803, could find no cause for the phenomenon.

The general mean of the deviations observed by myself in the north and south plane in the experiments referred to, experiments much more careful and extensive than those which any one else had made in this matter, was a southerly movement of about 0.005 cm. in a fall of about 23 m. The probable error was about 0.004 cm., and I should have regarded the case as practically closed in favor of the negative if my predecessors had not, almost without exception, reported a considerable southerly excursion. On the whole I was disposed to try the question further, and accordingly applied in 1904 for permission to make experiments for this purpose in the great monument at Washington, D. C., where a sheer fall of about 165 m. is possible. The monument is in the care of the War Department, and at first the authorities applied to acted favorably upon my petition. A few months later, and before I had made any overt preparations for the work proposed, some change of management or of mind occurred in the Department, and the permission previously granted me was courteously but firmly withdrawn, "for the reason that the monument was designed as a memorial to General Washington." I have long since come to

[1] Physical Review, 1903, **17**, 179 and 245; These Proceedings, 1904, **39**, 339.

the conclusion that this action was a fortunate one for me, as the investigation would certainly have been tedious and expensive and would probably have been inconclusive.

But the easterly deviation also was, incidentally, measured in my experiments at the Jefferson Laboratory, and the general mean value found for it was 0.149 cm., whereas the value given by the theoretical formula,

$$y = \frac{1}{3} gu \cos\lambda \times t^3,$$

where u is the angular velocity of the earth's rotation, λ is the latitude, and t is the time of fall in seconds, is 0.177 [2] cm. for the case in hand. The probable error of the observed general mean is perhaps greater than that for the southerly deviation, but is not great enough to account for the difference between the observed and the theoretical easterly value. I did not give in any of my previous papers on this subject the formula of Gauss or that of Laplace for the easterly deviation of a body falling in air, though I had given considerable attention to their treatment of the effect of air resistance, but closed my discussion of the matter thus: "The mean easterly deviation actually found in these experiments, 0.149 cm., differs 0.03 cm. from this theoretical value, — a quantity too large to be accounted for by the resistance of the air. I attach but little significance to this discrepancy, as the conditions for determining the easterly deviation in my work were plainly not so good as those for determining the southerly deviation."

Thus the matter stood till last April, when I received from Professor Hagen of the Vaticana Specola Astronomica the suggestion that I should make some experiments to find out how much the resistance of the air really amounted to, in order to see whether it might not after all go some distance toward explaining the discrepancy between the observed and the calculated easterly deviation. Father Hagen puts the statement of Gauss concerning the effect of air resistance so clearly, that I shall copy his words, changing, however, the nomenclature slightly. He writes:

"Gauss puts the height of the fall, determined by *linear* measure, $= f$, and $\frac{1}{2} gt^2 = f + \delta$, determined from the observed *time* of the fall. The difference δ is owing to the resistance of the air. Then

$$\text{Deviation } y = \tfrac{2}{3} \cos\lambda ut \left(f - \tfrac{1}{2}\delta\right)."$$

It was easy to carry out the suggestion thus given, and accordingly in October I reëstablished the releasing part of my apparatus at the top

[2] I have given this previously as 0.179, but 0.177 is more nearly correct.

of the Laboratory tower and had a new cloth tube suspended for the balls to drop through. This tube, like the old one, which had wasted away, was about 35 cm. in diameter, and the balls fell along its axis.

At the bottom of the tower the receiving apparatus was now a horizontal plate of brass, fastened at one end but free at the other, so as to be capable of up and down motion. Near the free end of this plate a square hole, about 5 cm. on each side, was cut. Over this hole was placed in some cases a sheet of lead somewhat narrower than the hole but long enough to be clamped fast to the brass plate at each end. Later a thin sheet of wood was placed over the hole before each fall. In either case the ball, after falling from the top of the tower, would strike the cover of the hole and break through it, the first shock of its impact pulling the brass plate down far enough to break the contact which made part of an electrical circuit including a chronograph. At the top of the tower the release of the ball broke the same electrical circuit, which was, however, closed a fraction of a second later. It is hardly necessary to give further details of the apparatus except this, that the chronograph, which was driven by an electric motor at the rate of about 3 cm. per second, was not under the best of control, and it was accordingly necessary to make a greater number of trials than would otherwise have been required in order to determine the time of fall with sufficient accuracy. It should be added that the rate of the clock giving the second signals at the chronograph was not very accurately known, as it varied somewhat from day to day, probably because of changes of temperature. Its error may have been as much as half a minute per day, but was probably less than this. An error of this magnitude is not serious for our present purpose, and the clock was in my calculations assumed to be correct.

On the 16th of October 17 balls were dropped with such success as to give usable records. The mean time of fall was 2.176 seconds, with a probable error about 0.002 second.

On the 25th of October I made another series of trials, dispensing with the protecting cloth tube. In this series records were obtained from 15 balls, the mean time of fall being 2.174 seconds, with a probable error about 0.004 second. It appears, then, that the presence of the tube has little if any influence on the time of fall.

The latitude of Cambridge being 42° 22′, very nearly, and the elevation above sea level very slight, we find that, according to the general formula for g as a function of λ, its value here is, to the first decimal place, 980.4. Accordingly we have as Gauss's $f+\delta$, the distance a body would fall in vacuum in 2.176 seconds,

$$f+\delta = \tfrac{1}{2} \times 980.4 \times 2.176^2 = 2321 \text{ cm.}$$

The distance f, the actual length of the fall, as measured by a steel tape which was tested by a Brown and Sharpe steel meter rod, was 2285 cm. Accordingly $\delta = 36$ cm., and the easterly deviation should be, according to Gauss,

$$y = \tfrac{2}{3} \cos 42^\circ\, 22' \times \frac{6.28}{86400} \times 2.176\,(2285 - 18) = 0.177 \text{ cm.},$$

that is, to the third place of decimals the value of the easterly deviation is not in our case affected by the resistance of the air, if I have correctly understood and used the formulas of Gauss.

Coefficient of Air Resistance.

It is perhaps worth while, since observations on the air resistance offered to the motion of spherical bodies are not over numerous, to work out from the data here at hand the coefficient of this resistance for the spheres here used, — bronze spheres, one inch in diameter, ground to a smooth surface, but left in a slightly greasy condition by their experience of being dropped into beds of tallow in their use six years ago.

The mere buoyant effect of air on bronze may properly be neglected in this discussion, as it is very small.

If we assume that the resistance of the air is proportional to the square of the velocity of the falling sphere, within the moderate range of velocity here considered, we have, as the net accelerating force on a ball of m grams, $(mg - kv^2)$ dynes, where k is the constant coefficient of resistance. Accordingly, writing c for $m \div k$, we find as the increment of velocity

$$dv = \left(g - \frac{v^2}{c}\right) dt, \tag{1}$$

whence

$$\frac{dv}{g - \frac{v^2}{c}} = \frac{c\,dv}{(gc - v^2)} = dt. \tag{2}$$

This equation, integrated for v between the limits 0 and v, and for t between the limits 0 and 2.176 (the observed value), gives

$$\frac{c}{2\sqrt{gc}} \left[\log \frac{\sqrt{gc} + v}{\sqrt{gc} - v}\right]_0^v = \tfrac{1}{2} \sqrt{\frac{c}{g}} \log \frac{\sqrt{gc} + v}{\sqrt{gc} - v} = 2.176. \tag{3}$$

We have further, if s is the distance fallen, from (2)

$$ds = vdt = \frac{vdv}{g - \frac{v^2}{c}}. \tag{4}$$

Integrating this equation for s between the limits 0 and 2285 (the observed value) and for v between 0 and v, we get

$$s = 2285 = -\frac{c}{2}\Big[\log\left(v^2 - gc\right)\Big]_0^v = -\frac{c}{2}\log\left(1 - \frac{v^2}{gc}\right). \tag{5}$$

Writing now (3) in the form

$$\frac{\sqrt{gc} + v}{\sqrt{gc} - v} = \epsilon^{4.352\sqrt{\frac{g}{c}}} \tag{6}$$

and (5) in the form

$$v = \sqrt{gc\left(1 - \epsilon^{-\frac{4570}{c}}\right)}, \tag{7}$$

and substituting for v in (6), we get

$$\frac{\sqrt{gc}\left(1 + \sqrt{1 - \epsilon^{-\frac{4570}{c}}}\right)}{\sqrt{gc}\left(1 - \sqrt{1 - \epsilon^{-\frac{4570}{c}}}\right)} = \epsilon^{4.352\sqrt{\frac{g}{c}}},$$

or

$$\frac{1 + \sqrt{1 - \epsilon^{-\frac{4570}{c}}}}{1 - \sqrt{1 - \epsilon^{-\frac{4570}{c}}}} = \epsilon^{4.352\sqrt{\frac{g}{c}}},$$

or

$$\frac{1 + \sqrt{1 - 10^{-\frac{1984.726}{c}}}}{1 - \sqrt{1 - 10^{-\frac{1984.726}{c}}}} = 10^{1.89005\sqrt{\frac{980.4}{c}}}. \tag{8}$$

The value of c which satisfies this equation I find to be about 48000. The value of k, the coefficient in question, is m, the mass of the ball, which is about 73.8 gm., divided by c.

$$k = 73.8 \div 48000 = 0.00154.$$

In Alger's "Exterior Ballistics" I find the following passage:

"Expressing the retardation caused by the resistance of the air in the form $A\frac{d^2}{w}v^n$, in which d is the diameter of the projectile in inches, w its weight in pounds and v its velocity in f. s., Mayevski's equations are as follows:"

"v between 790 f. s. and 0 f. s.,

$$\text{“}\frac{dv}{dt} = -A_7\frac{d^2}{w}v^2 \qquad \log A_7 = 5.669 \ldots (-10).$$

"The coefficient A depends on the shape of the projectile. In Mayevski's calculations the 'ogival' form [the shape of an ordinary artillery 'shell'] is assumed, the 'ogival' heads having two calibers radius. A would be greater with hemispherical heads."

Mayevski's formula is equivalent to

$$\text{Resistance (poundals)} = -w\frac{dv}{dt} = Ad^2 \times v^2.$$

Taking this formula for the case in which d is one inch, the diameter of the bronze balls, and the velocity is 1 cm. per second, we get for the "ogival" form,

$$\text{Resistance (poundals)} = A \div \overline{30.5}^2,$$

$$\text{“} \qquad \text{(dynes)} \qquad = A \div \overline{30.5}^2 \times (453 \times 30.5) = 0.00069.$$

This is about 45 per cent of the value, 0.00154, found above for k in the case of spherical one inch balls.

Jefferson Physical Laboratory,
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